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M. Sc. (Sem.-II) Examination September - 2025

Mathematics - Advanced Abstract Algebra (PGMTH : 201)

Time: 3 Hours] **[Total Marks: 70**

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) Attempt all questions. (3) Follow usual notations and conventions. (4) Use of non-programmable scientific calculator is permitted	Student's Signature

Q.1. Answer any TWO of the following: (14)

(a) Define conjugate of an element in a group. Prove that conjugacy is an equivalence relation.

(b) Describe the class equation of the group, by proving the necessary results.

(c) If $o(G) = p^n$, where p is prime, then prove that $o(Z(G)) > 1$.

Q.2. Answer any TWO of the following: (14)

(a) State and prove Sylow's first theorem.

(b) Let G be internal direct product of $N_1, N_2, \ldots, N_n$ and let $T = N_1 \times N_2 \times \ldots \times N_n$. Prove that G and T are isomorphic.

(c) Define primitive polynomial. If $f(x)$ and $g(x)$ are primitive polynomials, then prove that $f(x)g(x)$ is a primitive polynomial.

Q.3. Answer any TWO of the following: (14)

(a) State and prove the Eisenstein criterion for irreducibility of polynomials over the rationals.

(b) Define unique factorization domain. Prove that if R is a unique factorization domain, then so is $R[x]$.

(c) If the primitive polynomial $f(x)$ can be factored as the product of two polynomials having rational coefficients, then show that it can be factored as the product of two polynomials having integer coefficients.

Q.4. Answer any TWO of the following: (14)

(a) Define: Algebraic element. If K is a finite extension of F, prove that an element $a \in K$ is algebraic over F.

(b) If L is a finite extension of K and if K is a finite extension of F, then show that L is a finite extension of F, and $[L:F] = [L:K][K:F]$.

(c) If $p(x) \in F[x]$ and if K is an extension of F, then for any element $b \in K$, prove that $p(x) = (x - b)\, q(x) + p(b)$ where $q(x) \in K[x]$ and $\deg q(x) = \deg p(x) - 1$.

Q.5. Answer any TWO of the following: (14)

(a) Define Splitting field. Show that the splitting field for every polynomial exists.

(b) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor.

(c) Define: Fixed field of G, where G is the group of automorphisms of K. Show that the fixed field of G is a subfield of K.